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# TECHNICAL NOTE

D-942

FLUTTER AT VERY HIGH SPEEDS

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SUMMARY

L This paper is concerned with a discussion of some of the problems  
1 of flutter and aeroelasticity that are or may be important at high speeds.  
6 Various theoretical procedures for treating high Mach number flutter are  
4 reviewed. Application of two of these methods, namely, the Van Dyke  
5 method and piston-theory method, is made to a specific example and compared with linear two- and three-dimensional results. It is shown that the effects of thickness and airfoil shape are destabilizing as compared with linear theory at high Mach number. In order to demonstrate the validity of these large predicted effects, experimental flutter results are shown for two rectangular wings at Mach numbers of 6.86 and 3. The results of nonlinear piston-theory calculations were in good agreement with experiment, whereas the results of using two- and three-dimensional linear theory were not.

In addition, some results demonstrating the importance of including camber modes in a flutter analysis are shown, as well as a discussion of one case of flutter due to aerodynamic heating.

INTRODUCTION

This paper is concerned with some problems of flutter and aeroelasticity at very high flight speeds. For this purpose high speeds will be defined as starting in the Mach number range of 2.5 to 3.

Some of the problems which are or may be important at high speeds are discussed according to the forces in the aeroelastic problem - aerodynamic, structural, and inertial. Under the aerodynamic part are:

(a) Nonlinear effect of airfoil shape, thickness, and angle of attack: There appears to be a very large effect of these factors on the flutter speed, which is discussed subsequently.

(b) Effect of shocks: Information is lacking and this area requires some research effort.

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<sup>1</sup>Supersedes recently declassified NACA Research Memorandum L57D16a by Harry L. Runyan and Homer G. Morgan, 1957.

(c) Boundary layer and viscous effects: Here again information is lacking (of course, even in the low-speed case) but, with the thick boundary layers encountered at high-speed flight, the dynamics of the boundary layer could become important, particularly interactions of the boundary layer with shocks.

(d) Plan form: Some of the new plan forms having sweep angles of the order of  $75^\circ$  will pose special problems with respect to unsteady aerodynamics, and there arises the difficult problem of studying and developing theories that will take into account the effect of both airfoil shape and aspect ratio.

(e) Controls: Controls have always been a source of trouble for the flutter analysts. In the Mach number range of 10 to 20, the type of control that will prove to be satisfactory is not known. But from past experience, whatever type of aerodynamic control, if any, is found to be satisfactory, it will probably constitute a flutter problem.

Structures required for high-speed flight present another area of difficulty. Some of the problems are:

(a) Aerodynamic heating: An example of aerodynamic heating relating to flutter is briefly discussed.

(b) Panels and heat shields: For the flat-bottomed highly heated aircraft now envisioned for high-speed flight, it appears that the flutter of panels and heat shields will be a very real problem. Under high-temperature conditions, buckling will probably occur and will require nonlinear treatment.

(c) Plan form: For wings of high aspect ratio, the distortions of the wing involved mainly a twisting and bending of the wing, so that the elementary concepts of "beamology" could be used. However, the low-aspect-ratio wings now being considered behave more like plates and involve a large amount of chordwise deflection.

Inertial force is the third type of force in the aeroelastic problem. The aircraft structural weight is decreasing in comparison with the weight of the fuel, particularly with regard to missiles. Consequently, such nonlinear problems as fuel sloshing and swirl are becoming exceedingly important.

Three of these problems will be discussed: the effect of airfoil shape, structural plan form, and aerodynamic heating.

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## SYMBOLS

A	aspect ratio
a	speed of sound
b	half chord, in.
$C_p$	pressure coefficient, $\frac{p - p_o}{q}$
$I_\alpha$	moment of inertia about elastic axis
M	Mach number
m	mass of wing per unit of span length, $\frac{\text{lb-sec}^2}{\text{in.}^2}$
p	pressure at point x,y
$p_o$	pressure in undisturbed stream
q	dynamic pressure
$r_\alpha$	radius of gyration, $r_\alpha^2 = \frac{I_\alpha}{mb^2}$
t	thickness ratio
w	vertical induced velocity or downwash
V	stream velocity, ft/sec
x,y	Cartesian coordinates
$x_o$	pitch-axis location measured from leading edge, fraction of chord
$x_\alpha$	distance from pitch axis to center of gravity, positive rearward, fraction of semichord
$\gamma$	specific-heat ratio
$\omega_h$	bending frequency, radians/sec
$\omega_\alpha$	torsional frequency, radians/sec
$\omega_1$	first bending frequency, radians/sec

$\mu$  mass ratio,  $\frac{m}{\pi \rho b^2}$

$\rho$  fluid density

$\phi$  velocity potential

Subscripts:

L linear

NL nonlinear

## ANALYTICAL METHODS

This section is concerned with a brief description of the theoretical methods available for high Mach number studies. The complete nonlinear partial differential equations for the potential and the pressure coefficient are shown by

$$\phi_L + \phi_{NL} = 0 \quad (1)$$

$$C_p = C_{p,L} + C_{p,NL} \quad (2)$$

and can be broken down into a linear part plus a nonlinear part.

The linear solution now in general use is obtained from equations (1) and (2) by setting the nonlinear part equal to zero, as given by

$$\phi_L = 0 \quad (3)$$

$$C_p = C_{p,L} \quad (4)$$

and then solving the equations with suitable boundary conditions.

Several approximate methods are available for obtaining nonlinear solutions. The first of these is the solution of Van Dyke (ref. 1). He first eliminated the third-order terms from these two nonlinear equations (this procedure, in effect, eliminates the effect of finite shocks) and then inserted the solution for the linear equation (3) in the nonlinear part of equation (1). This procedure resulted in a linear partial differential equation plus a known function as

$$\phi_L + f(x,y,t) = 0 \quad (5)$$

$$C_p = C_{p,L} + C_{p,NL} \quad (6)$$

By a laborious technique, Van Dyke then solved these equations for the pressure on specific airfoils.

Another nonlinear method is the so-called piston theory. This procedure was originally suggested by Hayes (ref. 2), was used by Lighthill (ref. 3) to check the results of Van Dyke at high Mach number, and later was elaborated on and applied to the flutter problem by Ashley and Zartarian (ref. 4). The advantage of piston theory is its utter simplicity as compared with other theories. The pressure coefficient is easily derived on the basis of a piston moving in a one-dimensional channel. The expression for the pressure coefficient is given by

$$C_p = 2 \left[ \frac{1}{M} \left( \frac{w}{V} \right) + \frac{\gamma + 1}{4} \left( \frac{w}{V} \right)^2 + \frac{\gamma + 1}{12} M \left( \frac{w}{V} \right)^3 + \dots \right] \quad (7)$$

where  $w$  is the instantaneous vertical velocity of a point on the wing, and  $V$  is the stream velocity. Note that, as  $M$  is increased, the first term would become less important, and the higher order nonlinear terms would begin to take an added importance.

Another method is use of the Newtonian concept. In this procedure it is assumed that the flow striking the exposed surface is compressed to a very thin boundary layer and the force exerted on the airfoil is due to the component of momentum perpendicular to the surface. The resulting pressure coefficient

$$C_p = 2 \left[ \left( \frac{w}{V} \right)^2 - \frac{1}{3} \left( \frac{w}{V} \right)^4 + \dots \right] \quad (8)$$

was obtained by expanding  $\sin^2 \frac{w}{V}$ . For a curved surface or an oscillating surface, additional terms due to centrifugal force could be added. Note that the first term is missing as compared with piston theory and the coefficient of the squared term has a factor of 1 as compared with 0.6 for  $\gamma = 1.4$ . Later, use will be made of the Van Dyke and piston-theory solution.

#### APPLICATION TO SPECIFIC EXAMPLES

Some applications and comparisons of the various theories are given. In figure 1 are shown the results of calculating the flutter of a rectangular wing of panel aspect ratio 1.5 throughout the Mach number range of 1.3 to 10. The airfoil section was 65 series, tapering from 4 percent at the root to 3 percent at the tip. Four theories have been used: linear two-dimensional theory, linear three-dimensional theory, nonlinear

piston theory, and Van Dyke theory. The results are plotted against the stiffness-altitude parameter,  $\frac{b\omega_h}{a}\sqrt{\mu}$ . The flutter region is below the curves. Constant-altitude lines are horizontal and constant-dynamic-pressure lines are radial lines emanating from the origin. There are four points of interest in this plot. First, the large difference between the linear theories and the two theories which include the effect of thickness at the higher Mach numbers. This effect is primarily due to a forward shift in the center of pressure due to airfoil shape whereas the center of pressure for the two-dimensional linear theory is fixed at the 50-percent chord and the forward shift of the center of pressure in the three-dimensional tip does not predict as much forward movement as the nonlinear theories. Another point is the agreement of the more complicated Van Dyke theory with the simpler and more readily used piston theory at the higher Mach number. A third point of interest is the cross-over of the two- and three-dimensional theory at  $M = 1.6$ . It has usually been assumed that inclusion of the three-dimensional effects is a relieving effect when compared with the two-dimensional theory. This is usually true at the lower Mach numbers but it is not necessarily true at the high Mach numbers. A fourth point is that the effect of airfoil shape and thickness is destabilizing. For instance, at an altitude corresponding to a value of the ordinate of 3.3, nonlinear theory indicates that flutter would be experienced at a Mach number of 5, whereas the linear theory would predict the airfoil to be flutter free.

In figure 2 is shown the calculated effect of thickness and airfoil shape on flutter at  $M = 10$  obtained by using nonlinear piston theory. The stiffness-altitude parameter is shown plotted against the ratio of bending to torsion frequency. The flutter region is below the curve. Curves are presented for a flat plate, a 4-percent wedge, and a 4-percent biconvex airfoil. Curves for the flat plate or zero-thickness airfoil and the biconvex airfoil will now be considered. For low frequency ratio, the zero-thickness airfoil gives no flutter solution, whereas the biconvex airfoil shows a definite flutter solution. As the frequency ratio is increased, however, the curves tend to approach each other and at  $\frac{\omega_h}{\omega_a} = 1.2$  they actually cross. That is, the effect of

thickness is destabilizing for low values of the frequency ratio and stabilizing for high values, at least for this case. Note that the wedge has a shape similar to the flat plate except it is slightly destabilizing.

In figures 1 and 2 are shown some rather large and disturbing effects of thickness and airfoil shape in reducing the flutter speed. The question is then are these large effects, in fact, true. In an attempt to answer this question two wings have been fluttered at high speed. The frequency ratio selected for these wings was deliberately chosen so that as wide a spread as possible between the zero-thickness and the thickness solution could be obtained. For these cases the frequency ratio was

approximately 0.35. Although the parameters for figure 2 are not the same as those for the experiment, the trends are the same and at a frequency ratio of about 0.35 there is quite a difference between the zero-thickness and the thickness cases.

## COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS

### Flutter at Mach Numbers of 6.86 and 3.0

Results for two rectangular wings are shown in figures 3 and 4, each having a panel aspect ratio of 0.8. One is an 11-percent double-wedge section and the other is a 4-percent flat wing. The properties of these wings are given in the following table:

	11-percent wedge	4-percent plate
b . . . . .	2.55	2.57
m . . . . .	0.0001276	0.000127
$r_\alpha^2$ . . . . .	0.251	0.269
$x_0$ . . . . .	0.467	0.46
$x_\alpha$ . . . . .	0.0545	0.0745
$\omega_h$ . . . . .	110.9	106
$\omega_\alpha$ . . . . .	314	322
V at M = 3.0 . . .	2,110	2,120
V at M = 6.86 . . .	3,250	3,255

(The torsion mode for both wings was taken as unity across the span. The bending mode for the 11-percent wing was taken as  $f_h = 0.23 + 0.1925x$  and for the 4-percent wing as  $f_h = 0.335 + 0.186x$  where  $x$  varies from 0 to 4 inches.) The wings were very rigid and were mounted on flexible shafts so that, in effect, they corresponded to all-movable controls. The results are again plotted as the stiffness-altitude parameter against Mach number. The experimental results are shown as solid points and were obtained at Mach numbers of 6.86 and 3 in the Langley 11-inch hypersonic tunnel and the Langley 9- by 18-inch supersonic flutter tunnel, respectively. First, the double wedge will be examined. The solid line is the result of using nonlinear piston theory and fairly good agreement is indicated with the experiment. The two-dimensional, zero-thickness method gave no solution. The three-dimensional linear case indicated a flutter-free wing at a Mach number of 6.86 but gave a solution at a lower Mach number as indicated. For the 4-percent plate, similar agreement between piston theory and

experiment was obtained. Again, two-dimensional zero thickness gave no solution and indicated the wing to be flutter free; whereas, inclusion of the three-dimensional tip effect gave a solution as indicated. The value of reduced frequency  $k$  for the  $M = 6.86$  test was  $\frac{1}{70}$  and a first-order theory in frequency such as the piston theory should be satisfactory.

Thus, it appears that the detrimental effect of thickness on flutter as predicted by piston theory is in fact true and that nonlinear theories must be used at the high flight speeds. One interesting fact is that lines drawn through the experimental points intersect the origin; thus a constant "q" flutter variation is indicated.

#### Flutter of Delta Wings

Flutter calculations of two low-aspect-ratio cantilever wings will now be considered. In figure 5 the stiffness-altitude coefficient has been plotted against Mach number for  $45^\circ$  and  $60^\circ$  delta wings. The wings were flat plates with beveled leading and trailing edges. The circular points are the experimental results from reference 5 and the solid and dashed lines are analytical results. Piston theory was used for the aerodynamic input. A modal type of analysis which was based on experimentally measured mode shapes was used. Since these modes had a large amount of deflection in the chord direction, it did not seem that the deflection curves could be approximated by the usual procedure of bending and twisting of a straight line. Hence, analytical curves were fitted to the experimental deflection curves at each of 10 spanwise stations for use in the analysis. The results are shown by solid lines and show fairly good agreement with experiment. In order to assess the effect of chordwise deflection, the camber was arbitrarily eliminated from each mode and then recalculated. The results are shown by the dashed lines. For the  $60^\circ$  wing, the curve was shifted over to the nonconservative side, whereas for the  $45^\circ$  wing a very wide divergence is found. Thus the importance of including the camber deflection in the analysis of a low-aspect-ratio wing is demonstrated.

#### Flutter Due to Aerodynamic Heating

Another well-known problem of high speed is the effect of aerodynamic heating. With regard to flutter, the main effect of aerodynamic heating is to cause a loss in torsional stiffness, particularly during transient conditions. A solid duralumin wing has been tested at a Mach number of 2 in the preflight jet of the Langley Pilotless Aircraft Research Station at Wallops Island, Va. Two runs were made, a cold run during which the wing did not flutter and a hot run during which the wing fluttered. This phenomenon can be explained with the aid of figure 1. These

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calculations apply to this heated wing. In the cold condition the value of the stiffness-altitude parameter at a Mach number of 2 is 3.07 and is well in the stable region. During the fast start of the tunnel, the leading and trailing edges heated up much more rapidly than the thicker center section; this condition causes a momentary loss in torsional stiffness. Thus the torsional frequency was reduced; the stiffness-altitude parameter is correspondingly reduced and would follow a vertical line to an intersection of the flutter curve. Calculations of the loss in torsional stiffness have been made and show a reduction in the torsional frequency of 50 percent which is sufficient to intersect the flutter region. Thus, flutter which has been induced by aerodynamic heating, at least for a simple solid wing, can be calculated.

#### CONCLUDING REMARKS

New flutter and aeroelastic problems will appear at high flight speeds. Configurations dictated by high-speed requirements will probably also exhibit new problems in the lower speed ranges. An essential feature of many of these problems is their inherent nonlinearity. For accurate flutter prediction, inclusion of these nonlinearities, such as the effect of airfoil thickness and shape, is a necessity.

Langley Aeronautical Laboratory,  
National Advisory Committee for Aeronautics,  
Langley Field, Va., March 7, 1957.

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## FLUTTER FOR VARIOUS AERODYNAMIC THEORIES

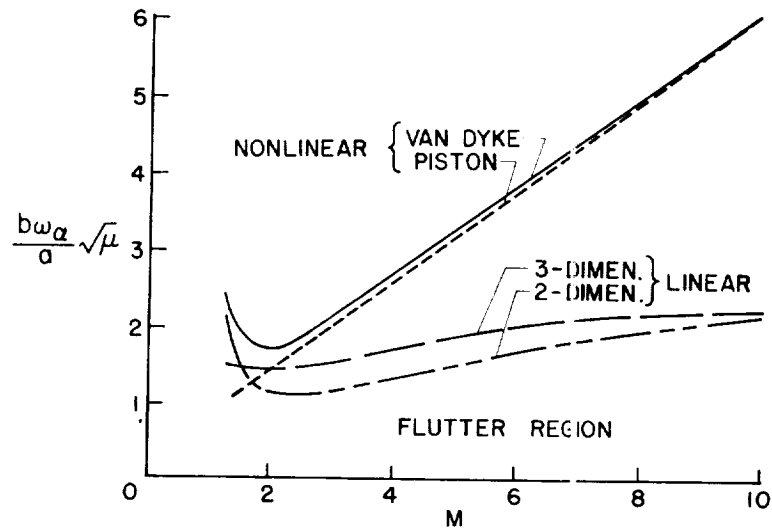


Figure 1

## CALCULATED EFFECT OF AIRFOIL SHAPE

$M=10; r_\alpha^2 = 0.25; C.G. = 50\%; E.A. = 4.0\%$

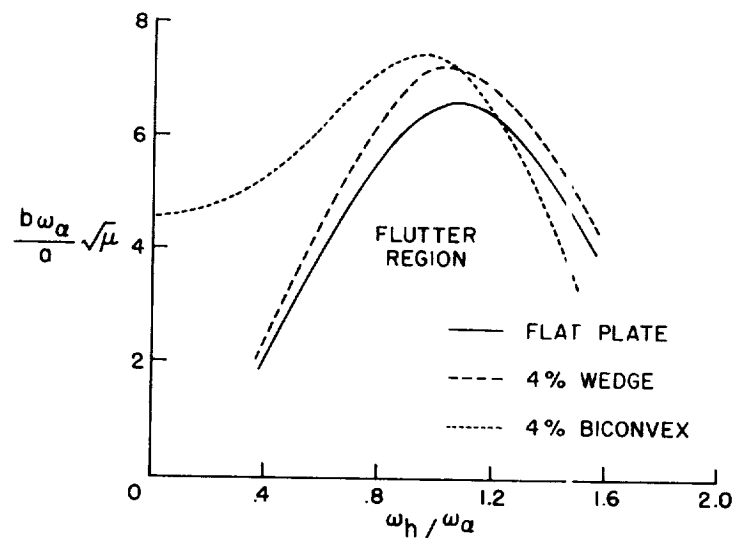


Figure 2

FLUTTER OF A DOUBLE-WEDGE WING AT HIGH SPEED  
E.A. = 46.7% ; C.G. = 49.4% ; A = 0.8

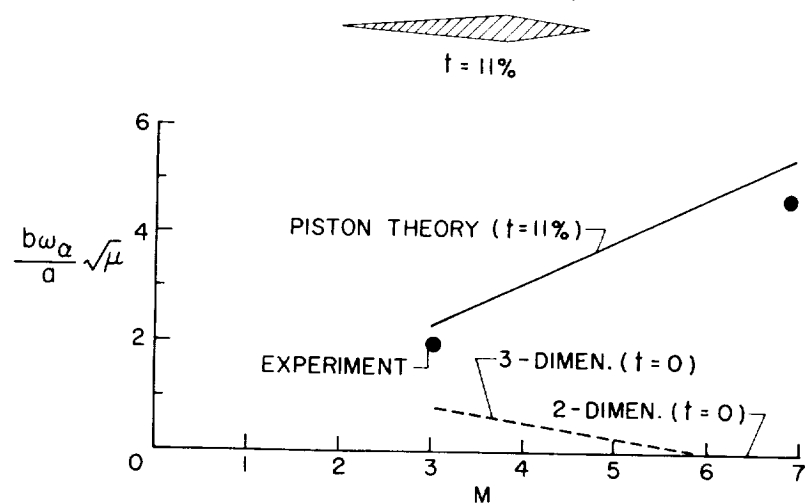


Figure 3

FLUTTER OF A THIN WING AT HIGH SPEED  
E.A. = 46% ; C.G. = 50% ; A = 0.8

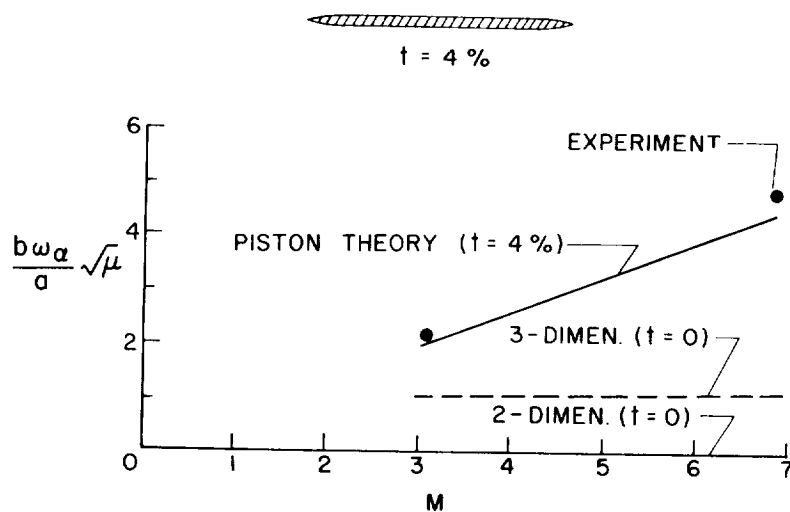


Figure 4

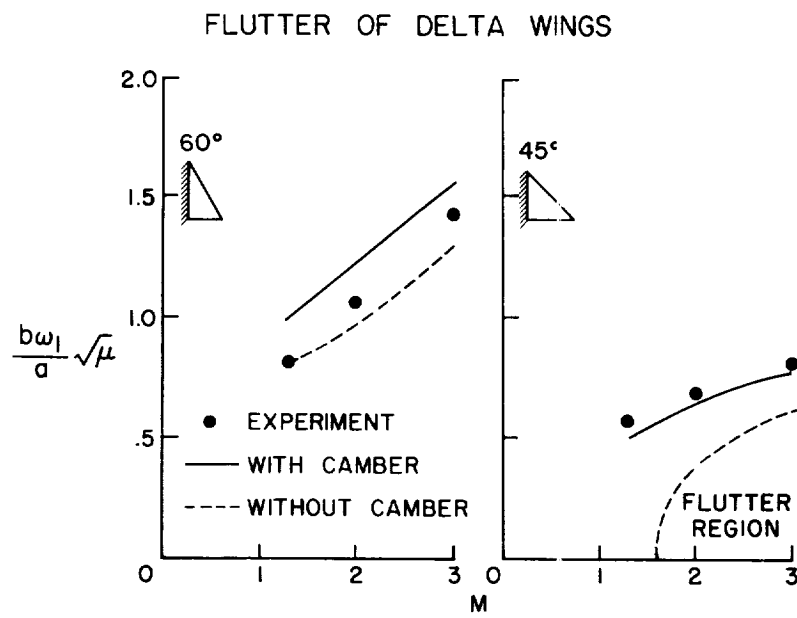


Figure 5